Network Transport Layer:
Network Resource Allocation Framework

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https://sngroup.org.cn/courses/cnns-xmuf25/index.shtml

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This deck of slides are heavily based on CPSC 433/533 at Yale University, by courtesy of Dr. Y. Richard Yang.

Outline

- Admin and recap
- Transport congestion control
  - what is congestion (cost of congestion)
  - basic congestion control alg.
  - TCP/Reno congestion control
  - TCP Cubic
  - TCP/Vegas
  - network wide resource allocation
    - general framework
    - objective function: axiom derivation of network-wide objective function
    - algorithm: general distributed algorithm framework
    - application: TCP/Reno TCP/Vegas revisited

Recap: TCP/Reno Throughput Modeling

$$\Delta W = \begin{cases} \frac{1}{W} & \text{if the packet is not lost} \\ -\frac{W}{2} & \text{if packet is lost} \end{cases}$$

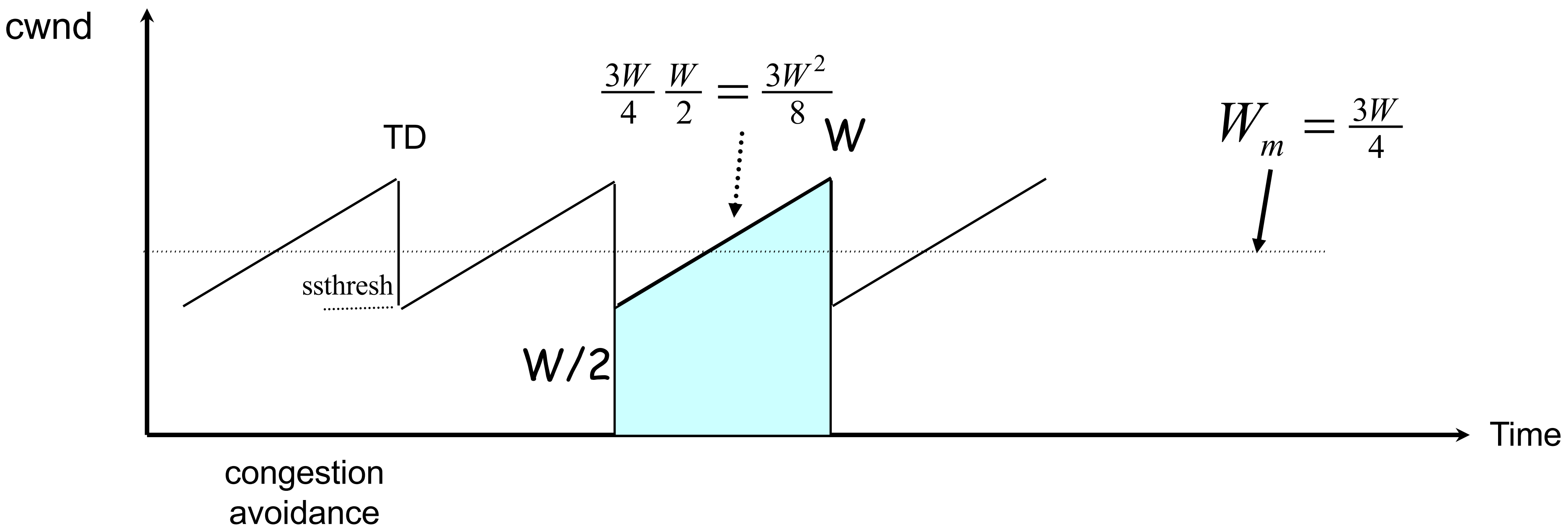
$$mean\ of\ \Delta W = (1-p)\frac{1}{W} + p(-\frac{W}{2}) = 0$$

=> $$mean\ of\ W = \sqrt{\frac{2(1-p)}{p}} \approx \frac{1.4}{\sqrt{p}}, \text{ when } p \text{ is small}$$

=> $$throughput \approx \frac{1.4S}{RTT\sqrt{p}}, \text{ when } p \text{ is small}$$

This is called the TCP throughput sqrt of loss rate law.

Recap: TCP/Reno Throughput Modeling



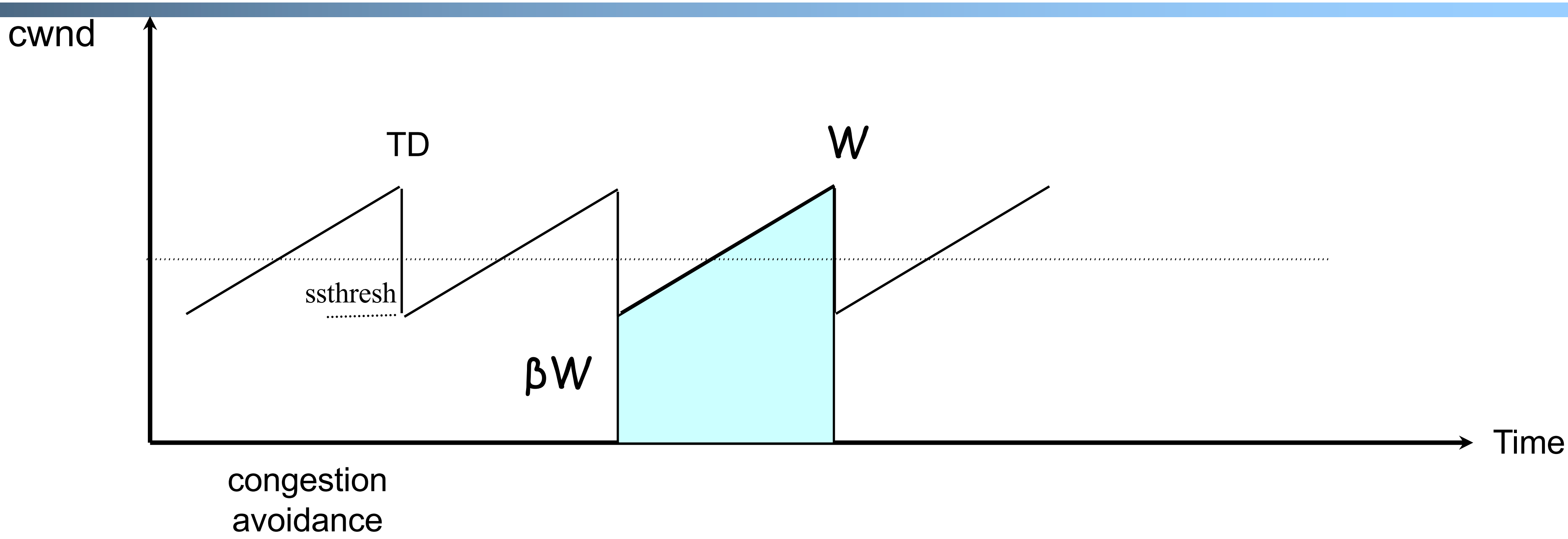
Total packets sent per cycle = (W/2 + W)/2 * W/2 = $3W^2/8$

Assume one loss per cycle => p = $1/(3W^2/8) = 8/(3W^2)$

$$\Rightarrow \quad W = \frac{\sqrt{8/3}}{\sqrt{p}} = \frac{1.6}{\sqrt{p}}$$

$$\Rightarrow throughput = \frac{S}{RTT}\frac{3}{4}\frac{1.6}{\sqrt{p}} = \boxed{\frac{1.2S}{RTT\sqrt{p}}}$$

Recap: Generic AIMD and TCP Friendliness



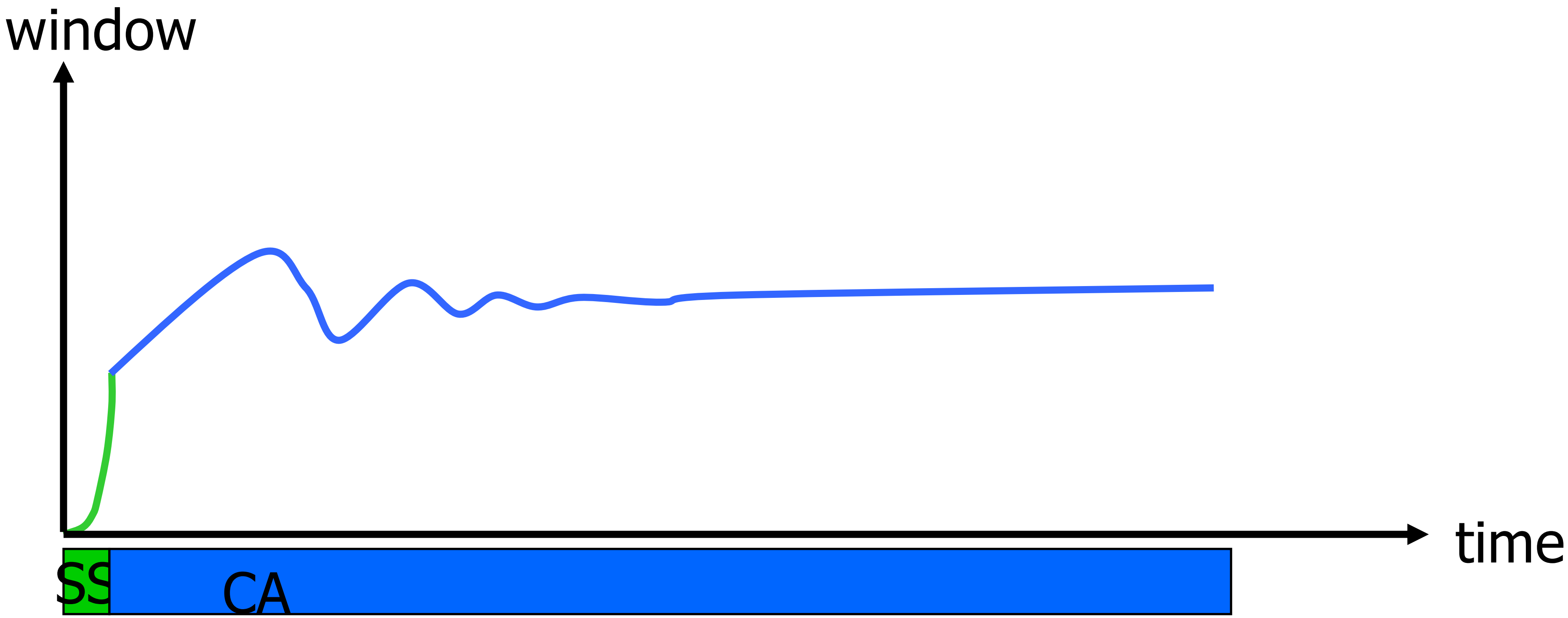
Total packets sent per cycle = $\frac{\beta W + W}{2} \frac{(1-\beta)W}{\alpha} = \frac{(1-\beta)(1+\beta)}{2\alpha} w^2$

Assume one loss per cycle $p = \frac{2\alpha}{(1-\beta)(1+\beta)w^2}$ $\quad w = \sqrt{\frac{2\alpha}{(1-\beta)(1+\beta)p}}$

$$\text{tput} = \frac{W_m S}{RTT} = \frac{S}{RTT} \frac{(1+\beta)W}{2} = \frac{S}{RTT} \sqrt{\frac{\alpha(1+\beta)}{2(1-\beta)p}}$$

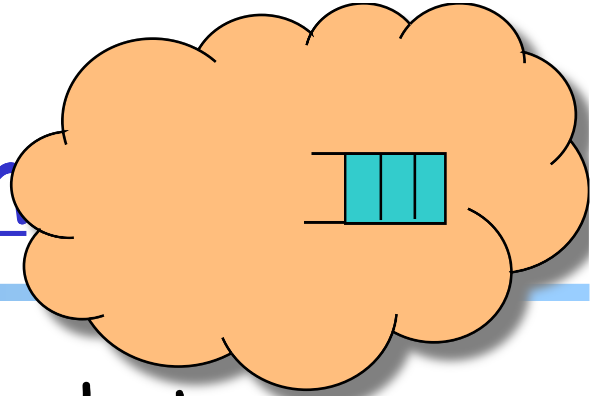
TCP friendly => $\alpha = 3 \frac{1-\beta}{1+\beta}$

TCP/Vegas (Brakmo & Peterson 1994)

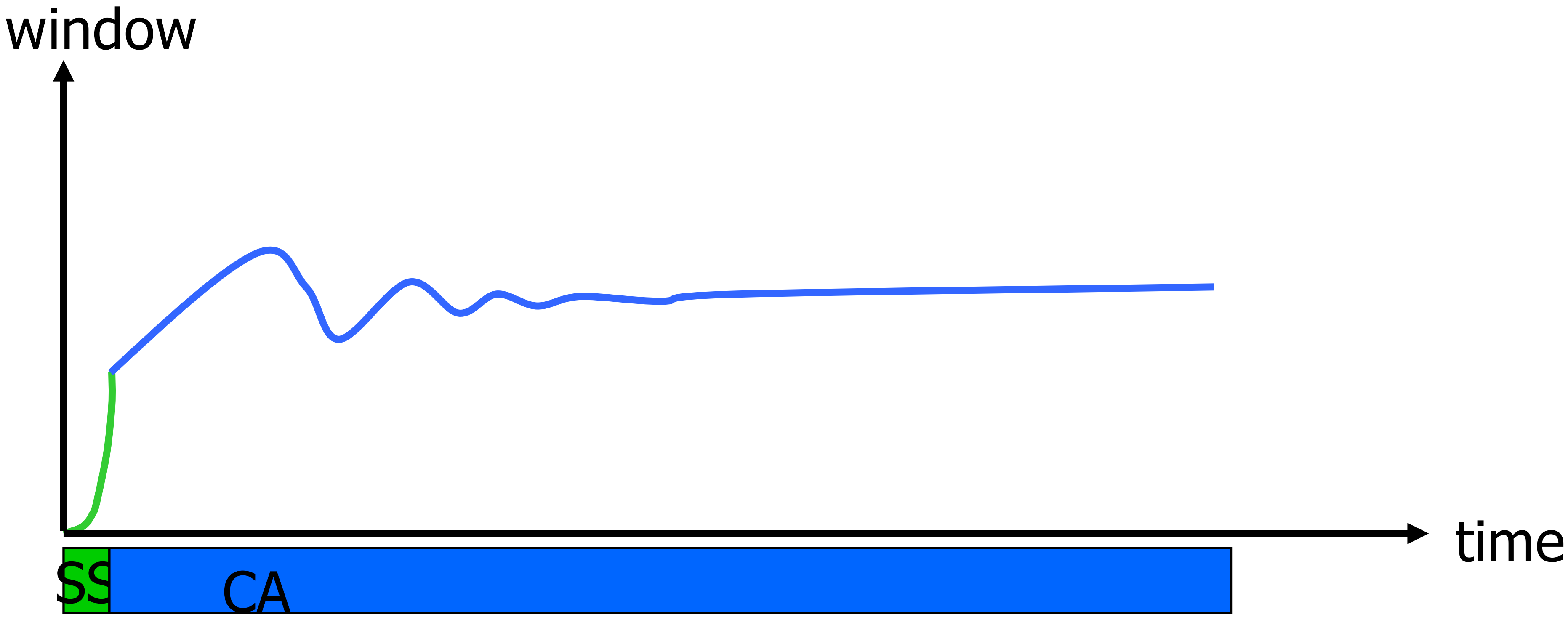


- Idea: try to detect congestion by delay before loss
- Objective: not to overflow the buffer; instead, try to maintain a *constant* number of packets in the bottleneck queue

TCP/Vegas: Key Question

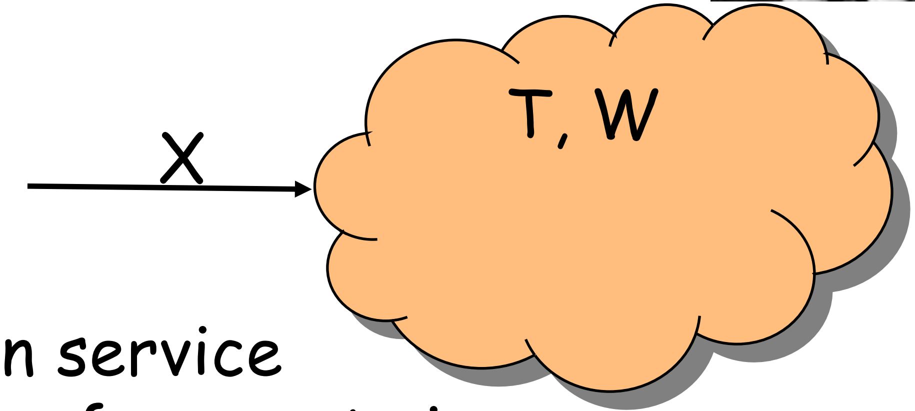


- How to estimate the number of packets queued in the bottleneck queue?



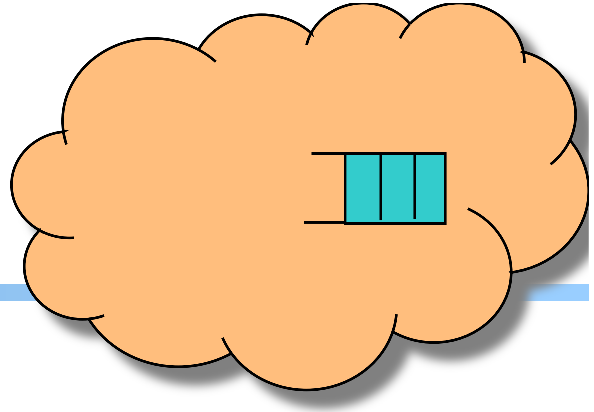
Recall: Little's Law

- For any system with no or (low) loss.
- Assume
  - mean arrival rate X, mean service time T, and mean number of requests in the system W
- Then relationship between W, X, and T:



$$W = XT$$

TCP/Vegas CA algorithm



$$T = T_{prop} + T_{queueing}$$

Applying Little's Law:

$$x_{vegas}\, T = x_{vegas}\, T_{prop} + x_{vegas}\, T_{queueing},$$

where $x_{vegas} = W / T$ is the sending rate

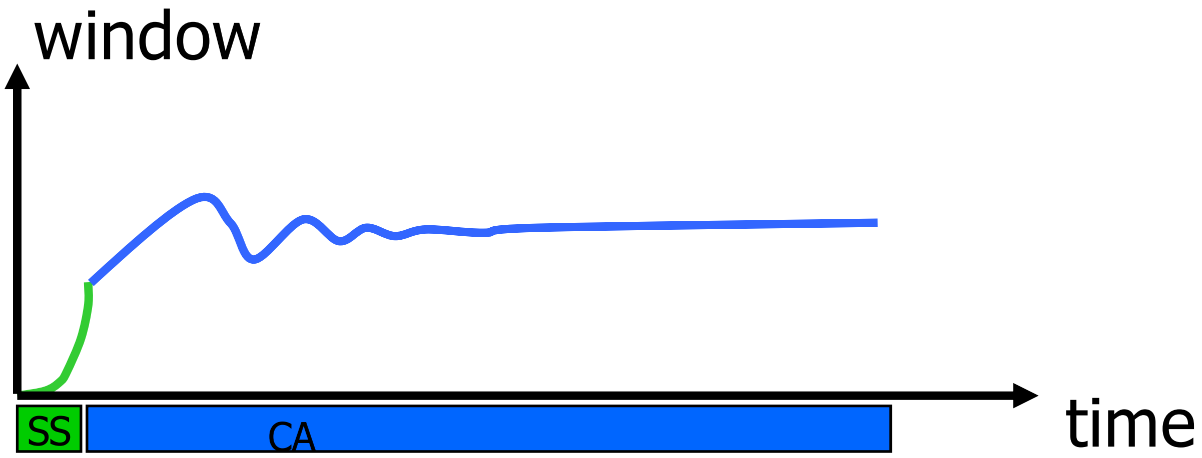
Then number of packets in the queue is

$$x_{vegas}\, T_{queueing} = x_{vegas}\, T - x_{vegas}\, T_{prop}$$

$$= W - W/T\, T_{prop}$$

TCP/Vegas CA algorithm

maintain a *constant* number of packets in the bottleneck buffer




```
for every RTT
{    if W – W/RTT RTTmin < α  then W ++
     if W – W/RTT RTTmin > α  then W --
}
for every loss
        W := W/2
```

queue size

Discussions

- If two flows, one TCP Vegas and one TCP reno run together, how may bandwidth partitioned among them?
- Issues that limit Vegas deployment?

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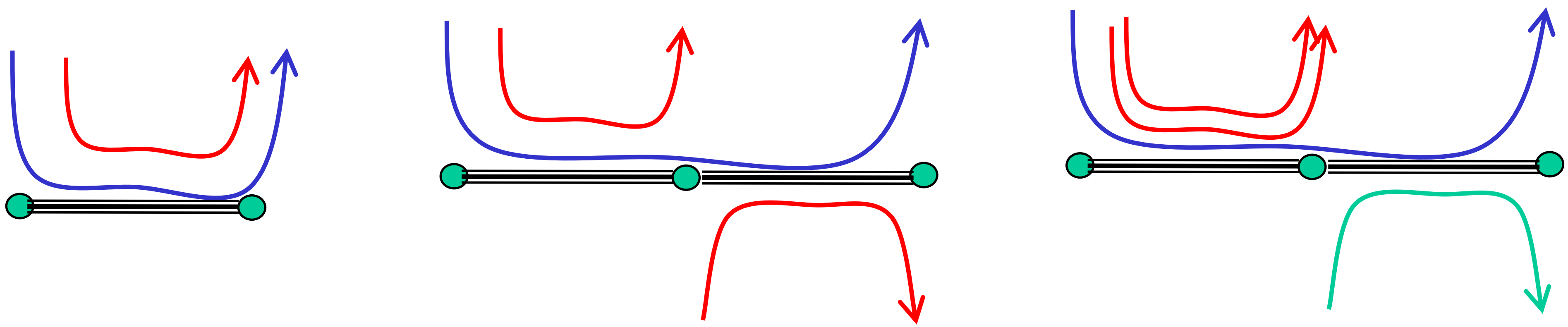
  - TCP/Vegas
  - network wide resource allocation
    - general framework

Motivation

- So far our discussion is implicitly on a network with a single bottleneck link; this simplifies design and analysis:
  - efficiency/optimality (high utilization)
    - fully utilize the bandwidth of the link
  - fairness (resource sharing)
    - each flow receives an ***equal*** share of the link's bandwidth

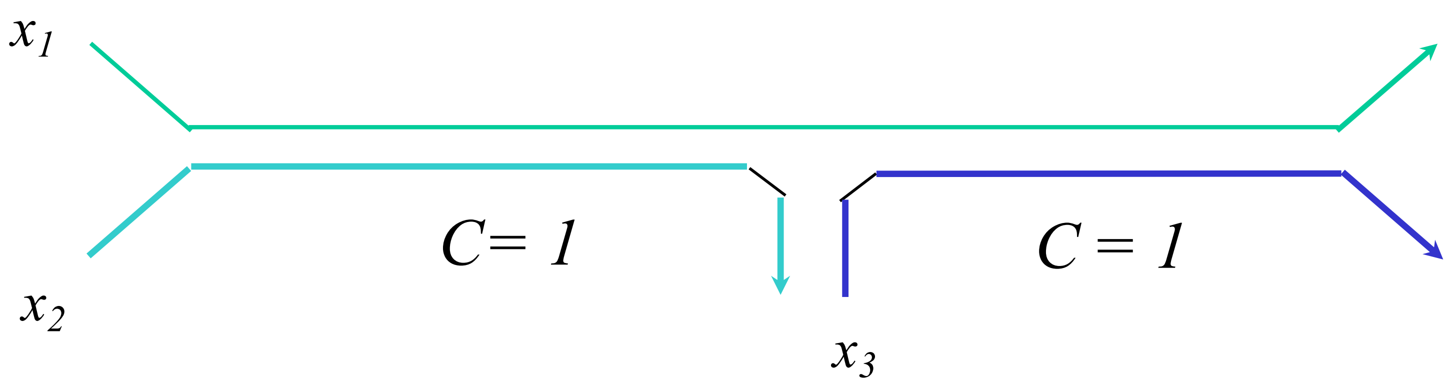
Network Resource Allocation

- It is important to understand and design protocols for a general network topology
  - how will TCP allocate resource in a general topology?
  - how should resource be allocated in a general topology?



Example: TCP/Reno Rates

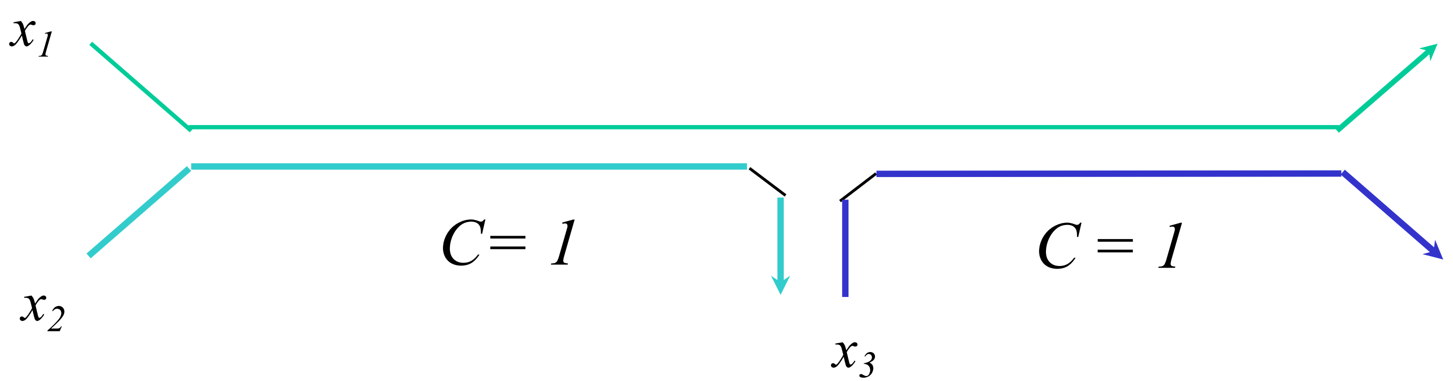
- Rates: $x_1 = \frac{1}{1+2\sqrt{2}} = 0.26$
  $x_2 = x_3 = 0.74$



Example: TCP/Vegas Rates

- Rates : $x_1 = 1/3$

  $x_2 = x_3 = 2/3$



Example: Max-min Fairness

- **Max-min fairness:** maximizes the throughput of the flow receiving the minimum (of resources)
  - Justification: John Rawls, *A Theory of Justice* (1971)
    - http://en.wikipedia.org/wiki/John_Rawls
  - This is a resource allocation scheme used in ATM and some other network resource allocation proposals

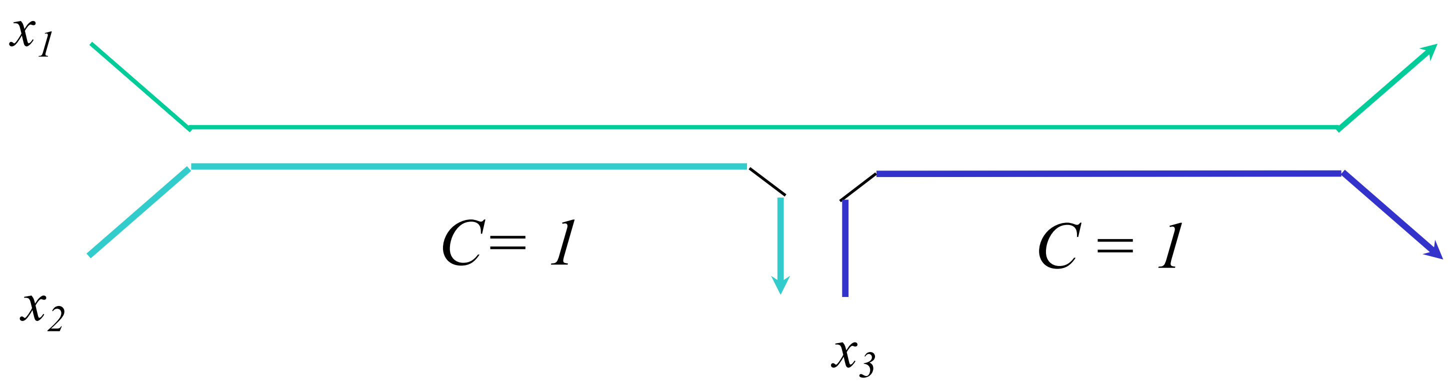
Example: Max-Min

$$\max_{x_f \geq 0} \quad \min\{x_f\}$$

$$\text{subject to} \quad x_1 + x_2 \leq 1$$

$$x_1 + x_3 \leq 1$$

- Rates: $x_1 = x_2 = x_3 = 1/2$



Framework: Network Resource Allocation Using Utility Functions

- A set of flows F
- Let x_f be the rate of flow f, and the utility to flow f is $U_f(x_f)$.
- Maximize aggregate utility, subject to capacity constraints

max	$\sum_{f \in F} U_f\left(x_f\right)$
subject to	$\sum_{f : f \text{ uses link } l} x_f \le c_l \text{ for any link } l$
over	$x \ge 0$

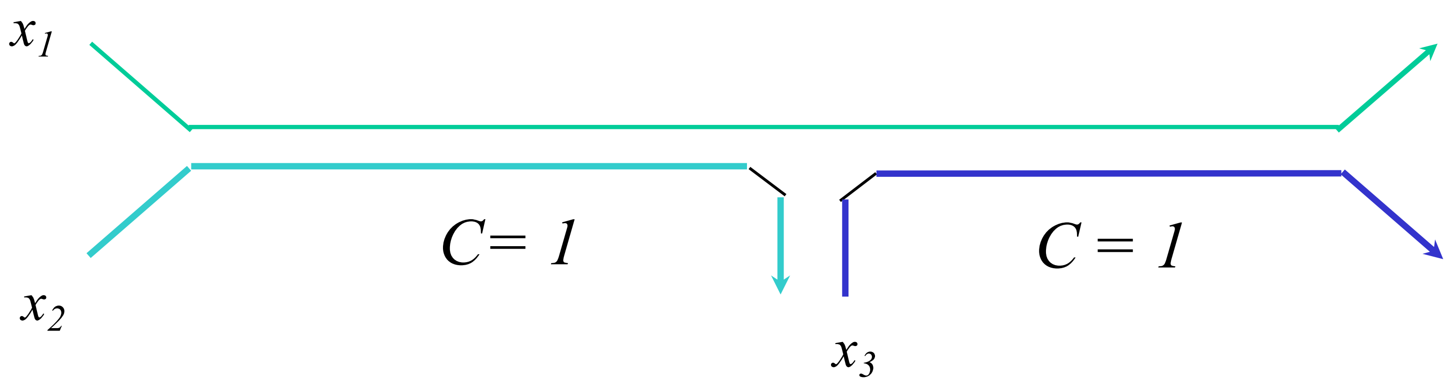
Example: Maximize Throughput

$$\max_{x_f \geq 0} \quad \sum_f x_f \qquad U_f(x_f) = xf$$

$$\text{subject to} \quad x_1 + x_2 \leq 1$$

$$x_1 + x_3 \leq 1$$

- Optimal: $x_1 = 0$
  $x_2 = x_3 = 1$



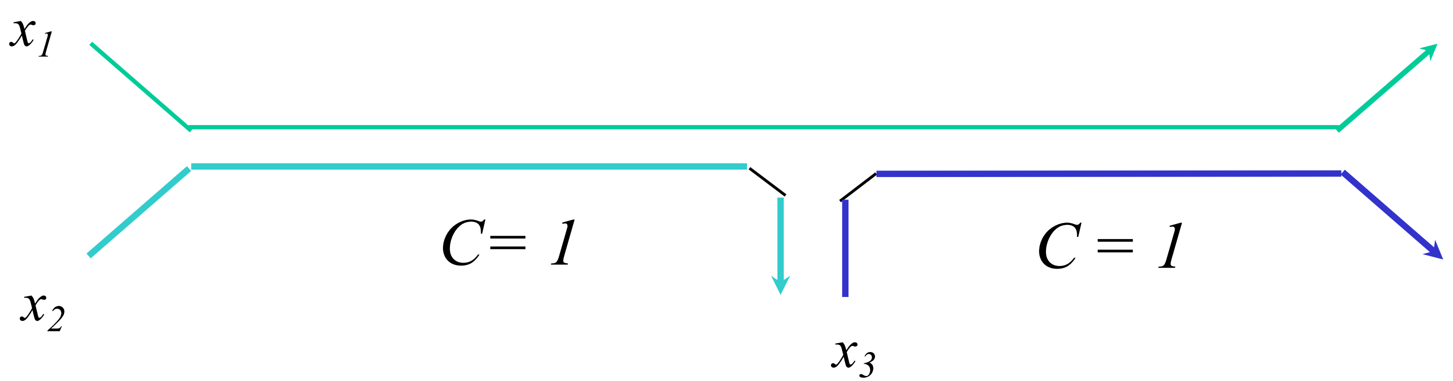
Example: Proportional Fairness

$$\max_{x_f \geq 0} \quad \sum_f \log x_f \qquad U_f\left(x_f\right) = \log\left(x_f\right)$$

$$\text{subject to} \quad x_1 + x_2 \leq 1$$

$$x_1 + x_3 \leq 1$$

- Optimal: $x_1 = 1/3$
  $x_2 = x_3 = 2/3$



Example 3: a "Funny" Utility Function

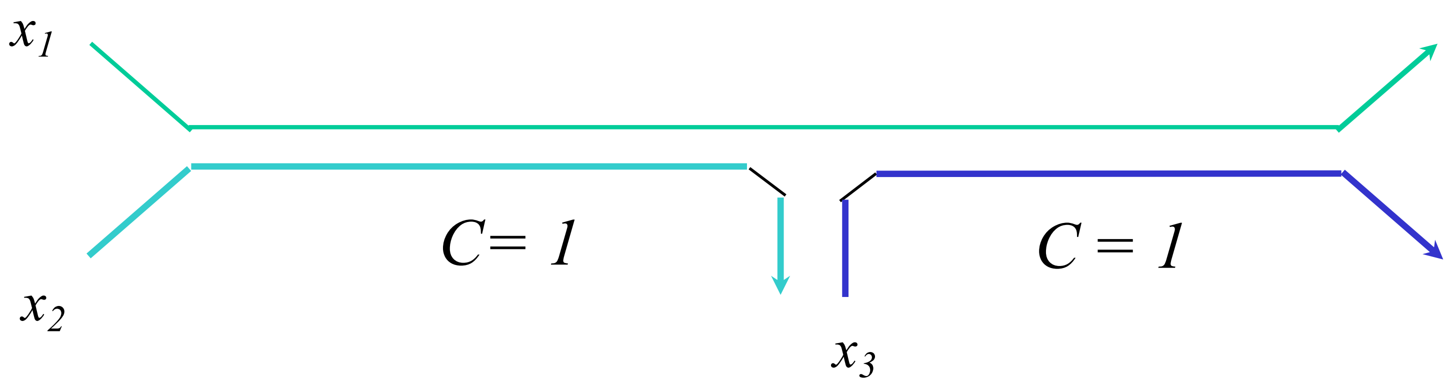
$$\max_{x_f \geq 0} \quad -\frac{1}{4x_1} - \frac{1}{x_2} - \frac{1}{x_3}$$

subject to

$$x_1 + x_2 \leq 1$$
$$x_1 + x_3 \leq 1$$

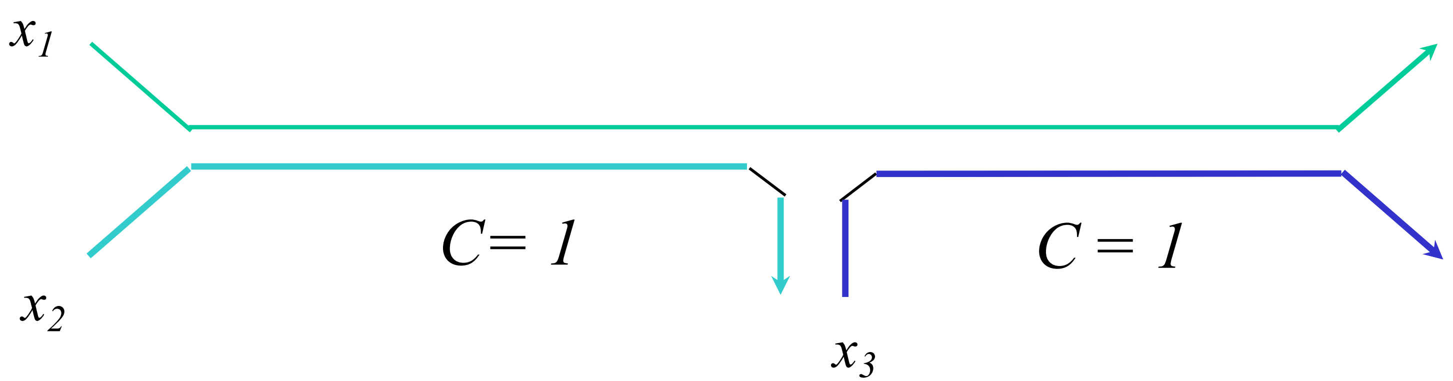
$$U_f(x_f) = -\frac{1}{RTT^2 x_f}$$

- Optimal: $x_1 = \frac{1}{1+2\sqrt{2}} = 0.26$
  $x_2 = x_3 = 0.74$



Summary: Allocations

Objective	Allocation (x1, x2, x3)		
TCP/Reno	0.26	0.74	0.74
TCP/Vegas	1/3	2/3	2/3
Max Throughput	0	1	1
Max-min	½	½	½
Max sum log(x)	1/3	2/3	2/3
Max sum of $-1/(RTT^2\ x)$	0.26	0.74	0.74



Questions

$$\begin{array}{ll} \max & \sum_{f \in F} U_f(x_f) \\ \text{subject to} & \sum_{f:f \text{ uses link } l} x_f \le c_l \text{ for any link } l \\ \text{over} & x \ge 0 \end{array}$$

- Forward engineering: systematically
  - design objective function
  - design distributed alg to achieve objective
- Science/reverse engineering: what do TCP/Reno, TCP/Vegas achieve?

Objective	Allocation (x1, x2, x3)		
TCP/Reno	0.26	0.74	0.74
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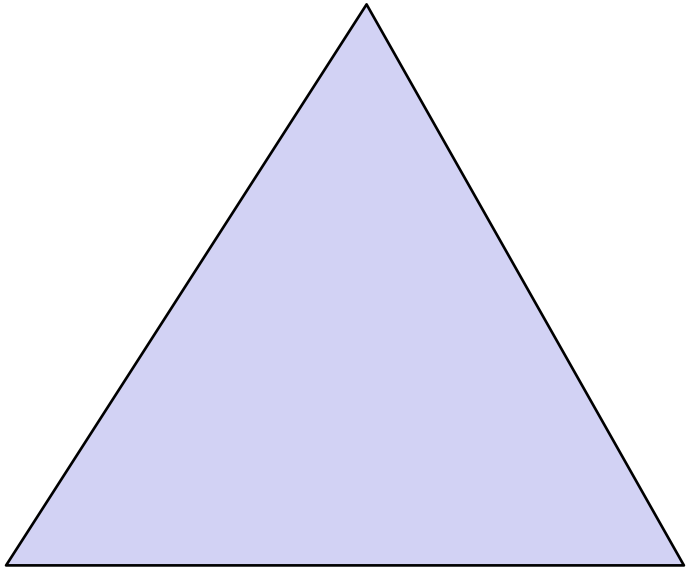
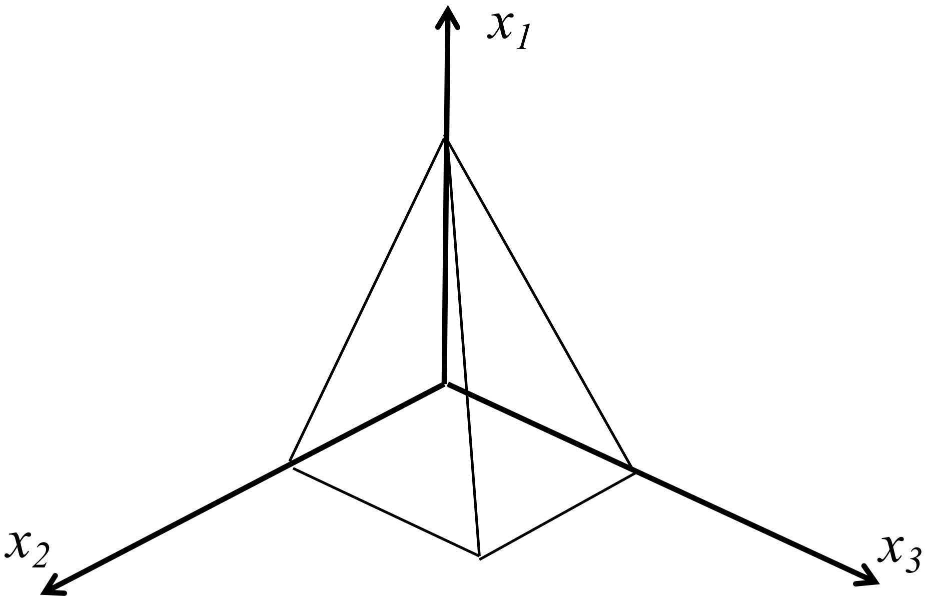
    - objective function: an example of an axiom derivation of network-wide objective function

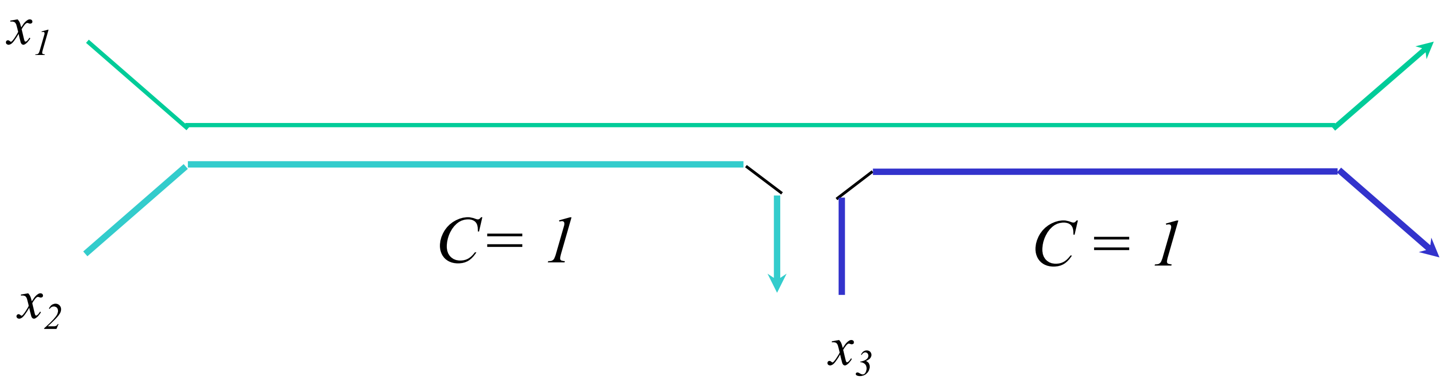
Network Bandwidth Allocation Using Nash Bargain Solution (NBS)



- High level picture
  - given the feasible set of bandwidth allocation, we want to pick an allocation point that is efficient and fair
- The determination of the allocation point should be based on "first principles" (axioms)

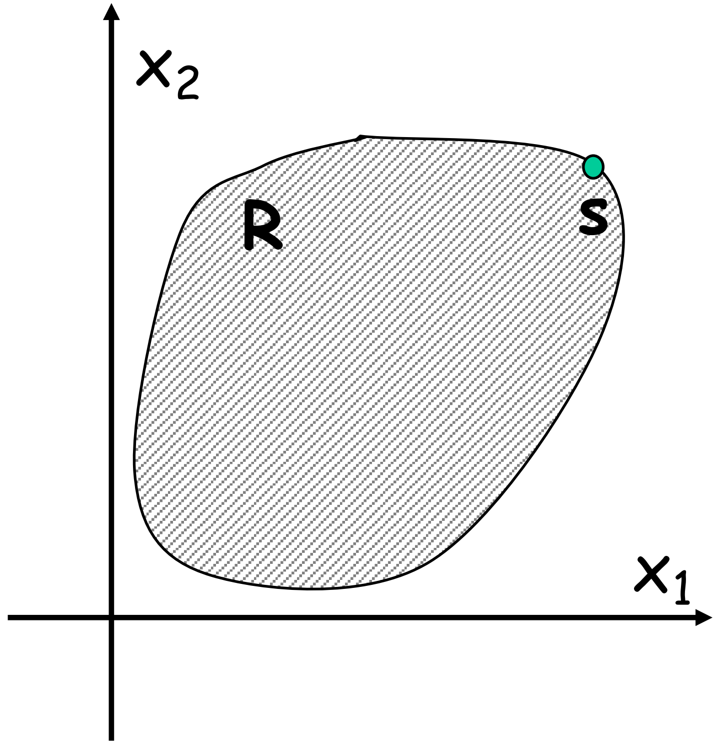
Network Bandwidth Allocation: Feasible Region





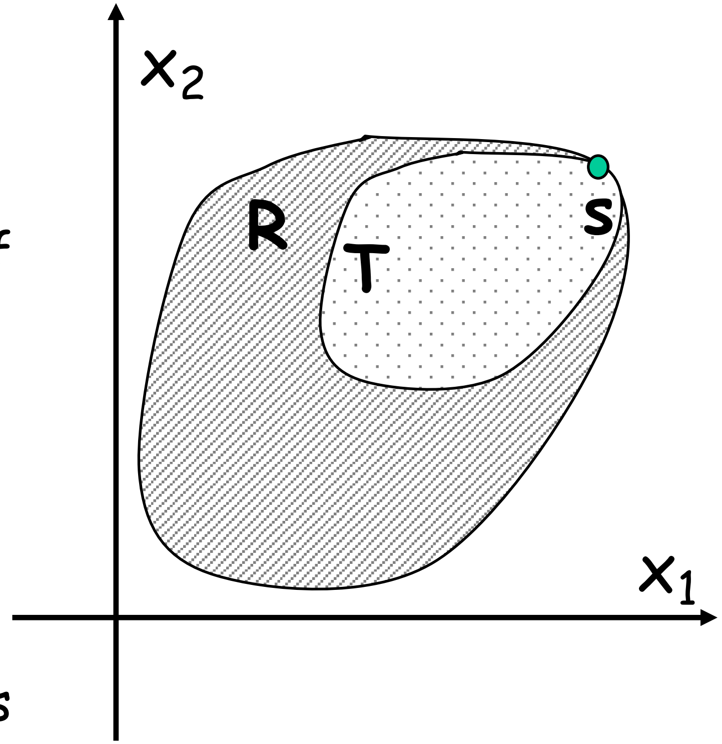
Nash Bargain Solution (NBS)

- Assume a finite, convex feasible set in the first quadrant
- Axioms



Nash Bargain Solution (NBS)

- Assume a finite, convex feasible set in the first quadrant
- Axioms
  - Pareto optimality
    - impossibility of increasing the rate of one user without decreasing the rate of another
  - symmetry
    - a symmetric feasible set yields a symmetric outcome
  - invariance of linear transformation
    - the allocation must be invariant to linear transformations of users' rates
  - independence of irrelevant alternatives
    - assume s is an allocation when feasible set is R, s $\in$ T $\subset$ R, then s is also an allocation when the feasible set is T



Nash Bargain Solution (NBS)

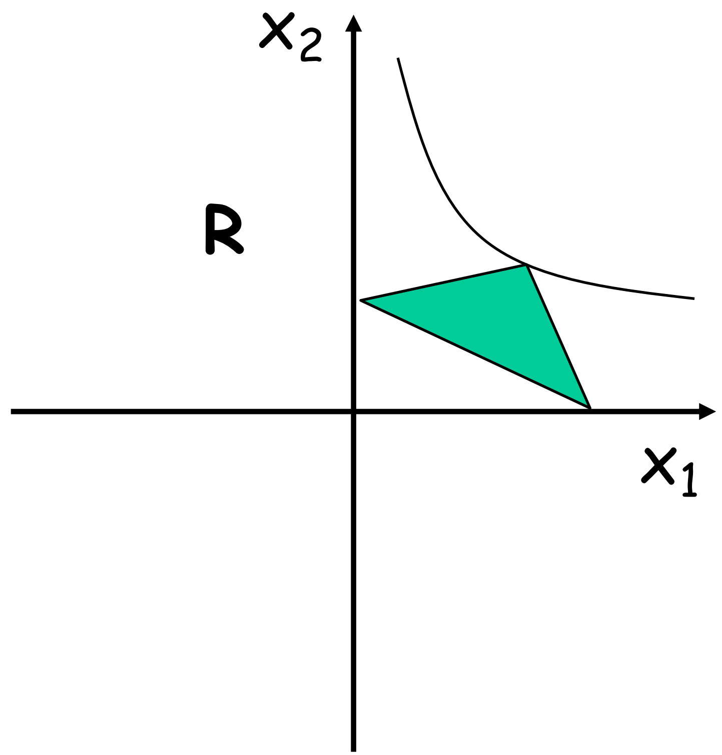
- Surprising result by John Nash (1951)
  - the rate allocation point is the feasible point which maximizes

$$x_1 x_2 \cdots x_F$$

- This is equivalent to maximize

$$\sum_f \log(x_f)$$

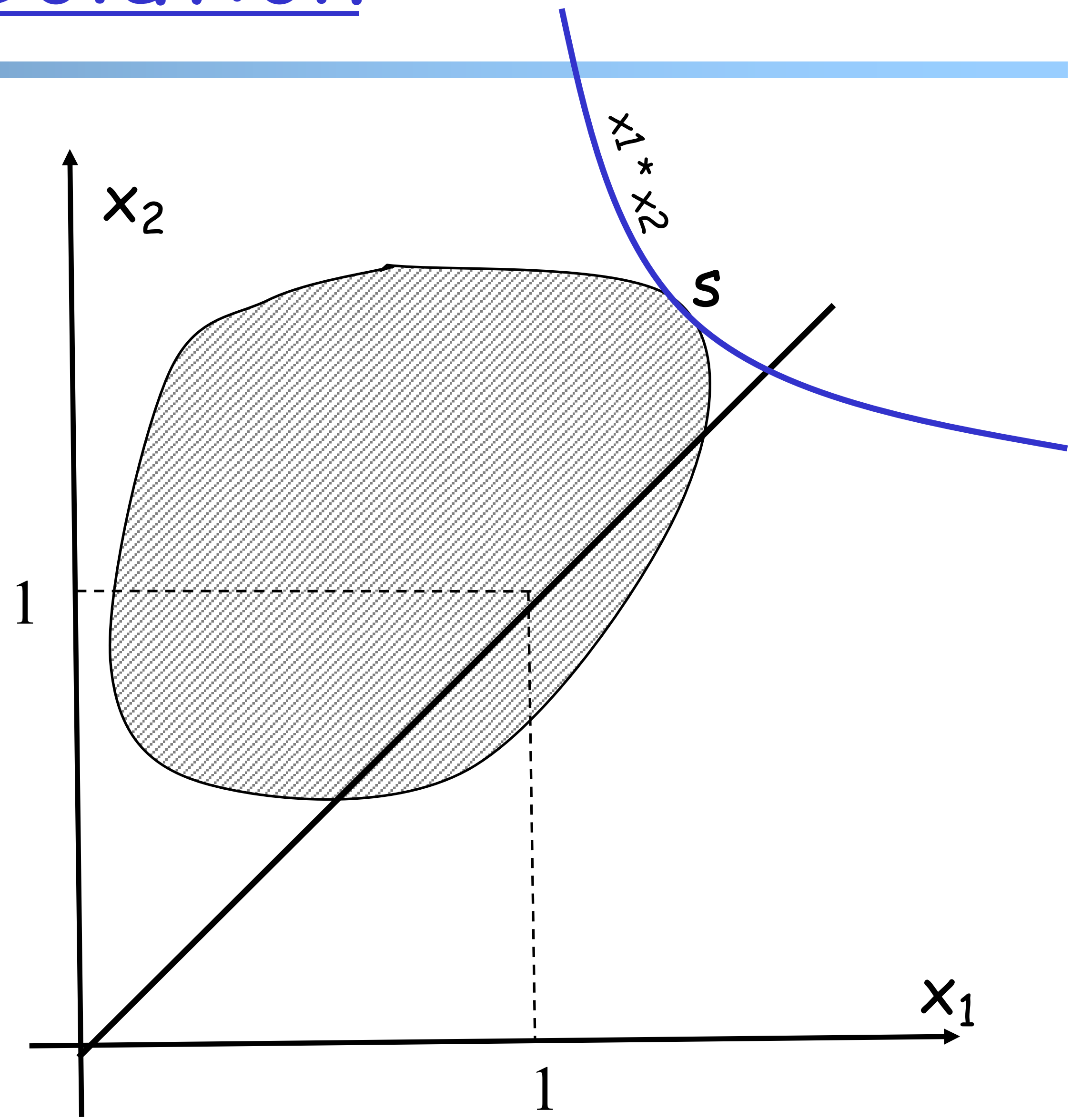
- In other words, assume each flow f has utility function $\log(x_f)$
- I will give a proof for F = 2
  - think about F > 2



Nash Bargain Solution

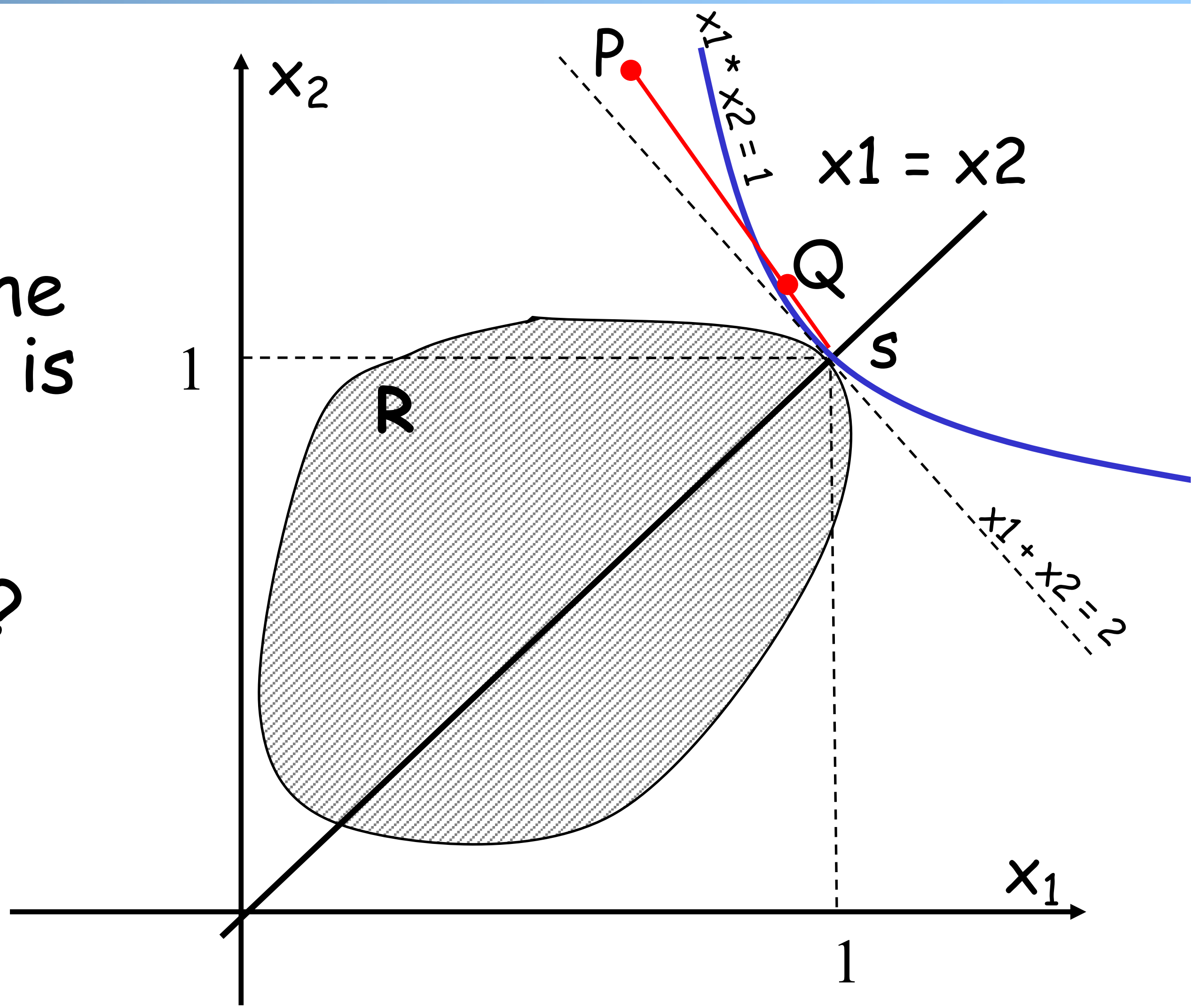
- Assume s is the feasible point which maximizes
x1 * x2

- Scale the feasible set so that s is at (1, 1)
  - how?



Nash Bargain Solution

Question: after the transformation, is there any feasible point with x1 + x2 > 2?

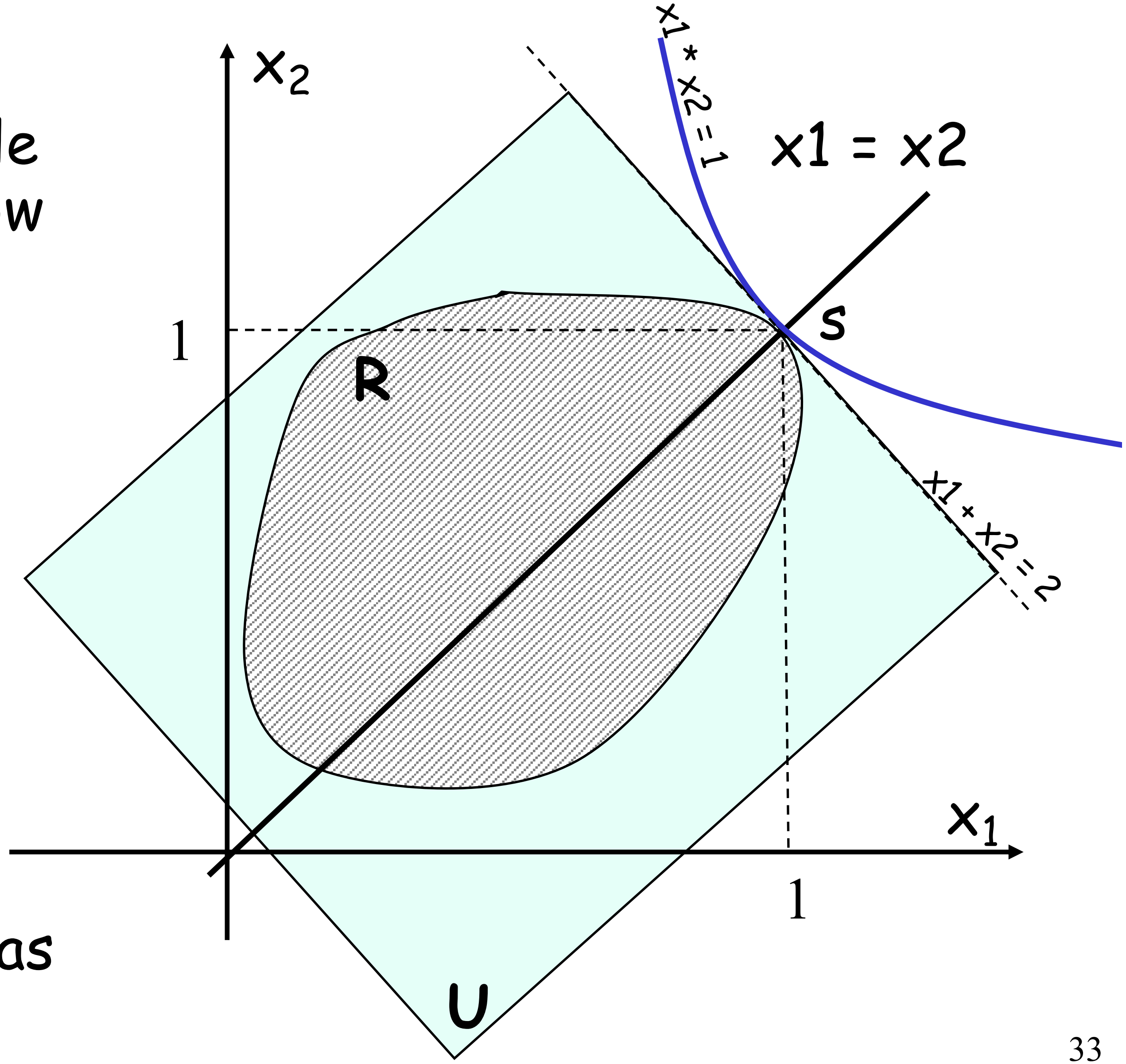


Nash Bargain Solution

- Consider the symmetric rectangle U containing the now feasible set

-> According to symmetry and Pareto, s is the allocation when feasible set is U

- According to independence of irrelevant alternatives, the allocation of R is s as well.



NBS $\Leftrightarrow$ Proportional Fairness

- Allocation is proportionally fair if for any other allocation, aggregate of proportional changes is non-positive, e.g. if x_f is a proportional-fair allocation, and y_f is any other feasible allocation, then require

$$\sum_f \frac{y_f - x_f}{x_f} \leq 0$$

Questions to Think

- Vary the axioms and see if you can derive any objective functions

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    - algorithm: general distributed algorithm framework
    - application: TCP/Reno TCP/Vegas revisited

Recall: Resource Allocation Framework

- The Resource-Allocation Problem:

$$\begin{array}{ll} \max & \sum_{f \in F} U_f\left(x_f\right) \\ \text{subject to} & \sum_{f: f \text{ uses link } l} x_f \le c_l \text{ for any link } l \\ \text{over} & x \ge 0 \end{array}$$

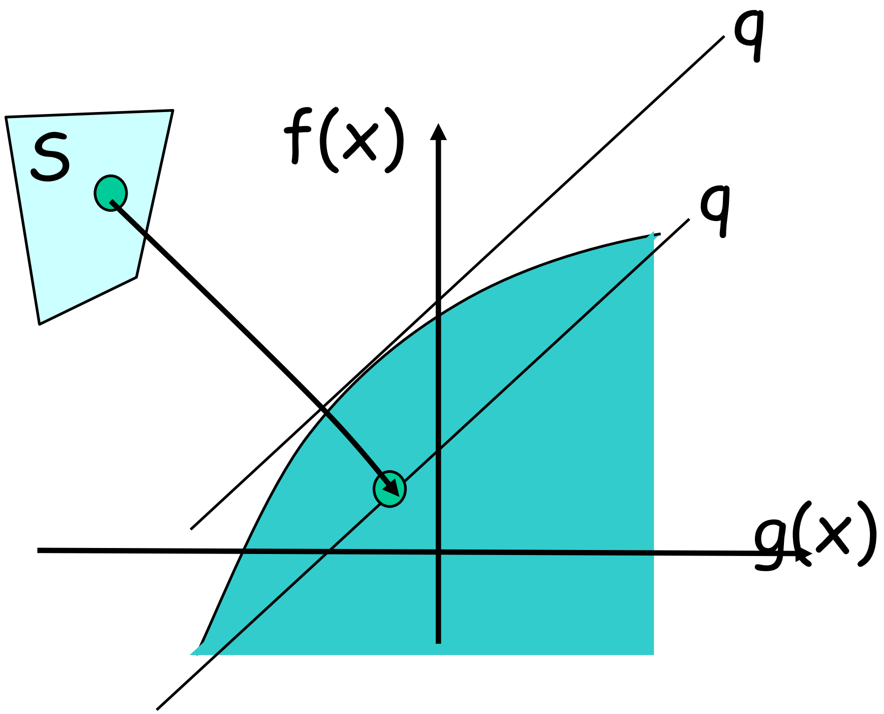
- Goal: Design a distributed alg to solve the problem.
- Discussion:
  - What are typical approaches to solve optimization, e.g.,?
    max U(x)
  - Why is the Resource-Allocation problem hard to solve by a distributed algorithm?

A Two-Slide Summary of Constrained Convex Optimization Theory

$$\begin{array}{ll} \max & \sum_{f \in F} U_f\left(x_f\right) \\ \text{subject to} & Ax \leq C \\ \text{over} & x \geq 0 \end{array}$$

$$\begin{array}{ll} \max & f(x) \\ \text{subject to} & g(x) \leq 0 \\ \text{over} & x \in S \end{array}$$

f(x) concave
g(x) linear
S is a convex set



-Map each x in S, to [g(x), f(x)]

-Top contour of map is concave

-Easy to read solution from contour

-For each slope q (>=0), computes f(x) – q g(x) of all mapped [f(x), g(x)]

$$D(q) = \max_{x \in S}\left(f(x) - qg(x)\right)$$

A Two-Slide Summary of Constrained Convex Optimization Theory

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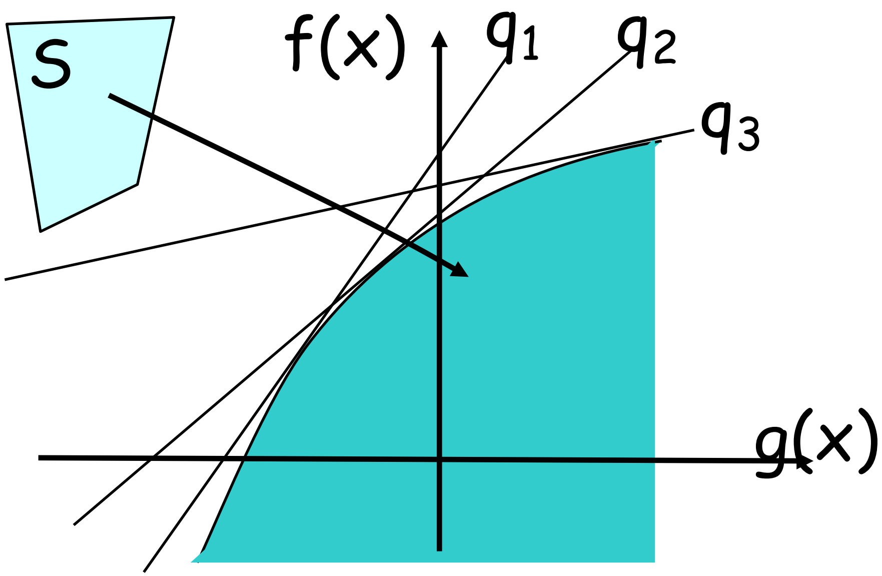
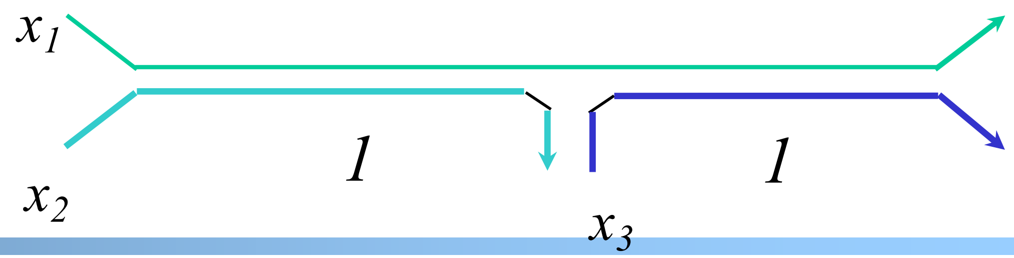


$$D(q) = \max_{x \in S} \left(f(x) - q g(x) \right)$$

-D(q) is called the dual;
q (>= 0) are called prices in economics

-D(q) provides an upper bound on obj.
- According to optimization theory:
when D(q) achieves minimum over all q (>= 0), then the optimization objective is achieved.

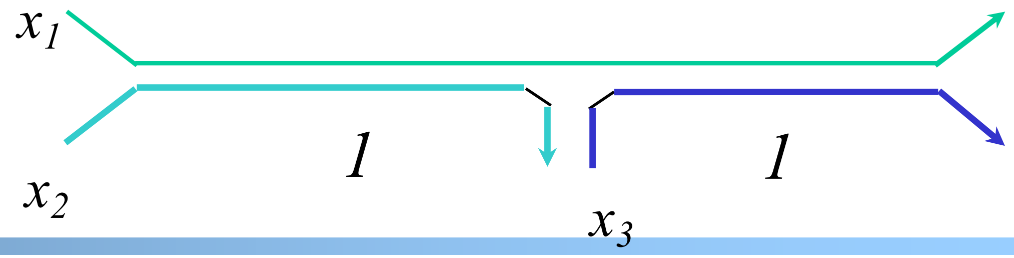
Dual of the Primal



$$\begin{array}{ll} \max & \sum_{f \in F} U_f(x_f) \\ \text{subject to} & \sum_{f: f \text{ uses link } l} x_f \le c_l \text{ for any link } l \\ \text{over} & x \ge 0 \end{array}$$

$$D(q) = \max_{x_f \ge 0} \left(\sum_f U_f(x_f) - \sum_l q_l \Big(\sum_{f:\text{uses } l} x_f - c_l \Big) \right)$$

Dual of the Primal



$$D(q) = \max_{x_f \geq 0} \left(\sum_f U_f(x_f) - \sum_l q_l \big(\sum_{f\text{:uses } l} x_f - c_l \big) \right)$$

$$= \max_{x_f \geq 0} \sum_f \left(U_f(x_f) - x_f \sum_{l\text{: f uses } l} q_l \right) + \sum_l q_l c_l$$

$$= \sum_f \max_{x_f \geq 0} \left(U_f(x_f) - x_f \sum_{l\text{: f uses } l} q_l \right) + \sum_l q_l c_l$$

Distributed Optimization: User Problem

- Given p_f (=sum of dual var q_l along the path) flow f chooses rate x_f to maximize:

$$\max_{x_f} \quad U_f(x_f) - x_f p_f$$
$$\text{over} \quad x_f \geq 0$$

- Using the price signals, the optimization problem of each user is independent of each other!

Distributed Optimization: User Problem

$$\max_{x_f} \quad U_f(x_f) - x_f p_f$$
$$\text{over} \quad x_f \geq 0$$

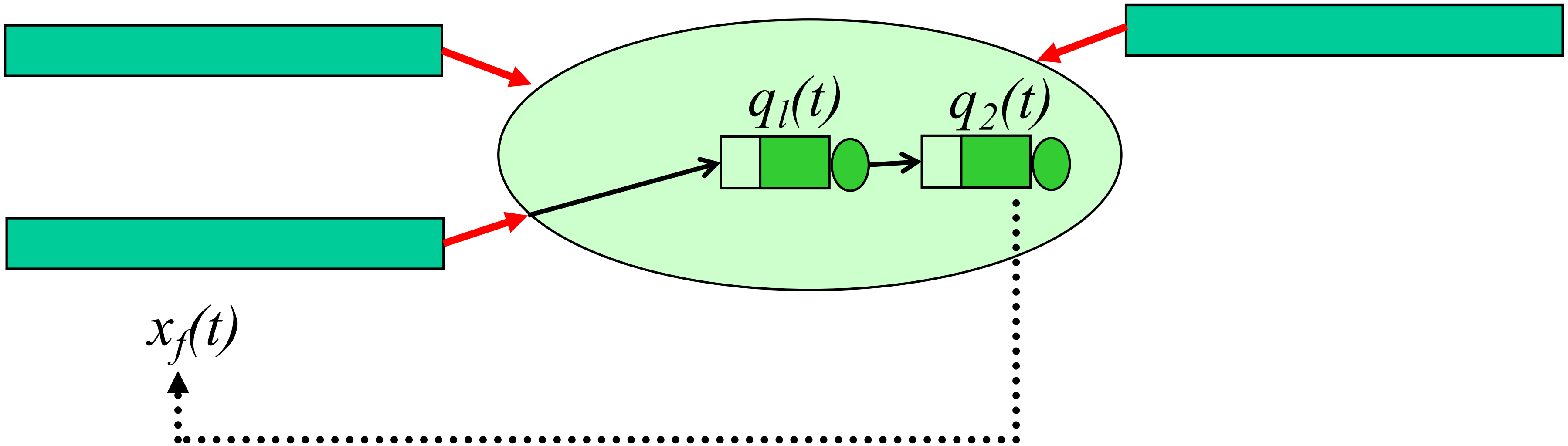
How should flow f adjust x_f locally?

$$\Delta x_f \propto U'_f(x_f) - p_f$$

At equilibrium (i.e., at optimal), x_f satisfies:

$$U'_f(x_f) - p_f = 0$$

Interpreting Congestion Measure



$$p_f = \sum_{\text{f uses l}} q_l$$

$$\Delta x_f \propto U'_f\left(x_f\right) - p_f$$

Distributed Optimization: Network Problem

$$D(q) = \sum_f \max_{x_f \ge 0} \left(U_f(x_f) - x_f \sum_{l: \text{f uses } l} q_l \right) + \sum_l q_l c_l$$

The network (i.e., link l) adjusts the link signals q_l (assume after all flows have picked their optimal rates given congestion signal)

$$\min_{q \ge 0} \tilde{D}(q) = \sum_l q_l (c_l - \sum_{f: \text{f uses } l} x_f)$$

Distributed Optimization: Network Problem

$$\min_{q \ge 0} D(q) = \sum_l q_l (c_l - \sum_{f:\text{ f uses } l} x_f)$$

how should link l adjust q_l locally?

$$\Delta q_l \propto -\frac{\partial D(q)}{q_l}$$

$$\frac{\partial}{\partial q_l} D(q) = c_l - \sum_{f:\text{uses } l} x_f$$

$$\Delta q_l \propto \sum_{f:\text{uses } l} x_f - c_l$$

System Architecture

- SYSTEM(U):

$$\begin{aligned} &\text{max} && \sum_{f \in F} U_f(x_f) \\ &\text{subject to} && \sum_{f : f \text{ uses link } l} x_f \le c_l \text{ for any link } l \\ &\text{over} && x \ge 0 \end{aligned}$$

- USER$_f$:

$$\Delta x_f \propto U'_f(x_f) - p_f$$

$$\begin{aligned} &\max_{x_f} && U_f(x_f) - x_f p_f \\ &\text{over} && x_f \ge 0 \end{aligned}$$

- NETWORK:

$$\Delta q_l \propto -\frac{\partial D(q)}{q_l}$$

$$\min_{q \ge 0} \widetilde{D}(q) = \sum_l q_l (c_l - \sum_{f : \text{f uses } l} x_f)$$

Decomposition Theorem

- There exist vectors **p** , **w** and **x** such that
  1. $w_f = p_f x_f$ for $f \in F$
  2. w_f solves $USER_f(U_f; p_f)$
  3. x solves NETWORK(w)

- The vector x then also solves SYSTEM(U).

Outline

- Admin and recap
- Transport congestion control
  - what is congestion (cost of congestion)
  - basic congestion control alg.
  - TCP/Reno congestion control
  - TCP Cubic
  - TCP/Vegas
  - network wide resource allocation
    - general framework
    - objective function: an example axiom derivation of network-wide objective function
    - algorithm: a general distributed algorithm framework
    - application: TCP/Reno and TCP/Vegas revisited

TCP/Reno Dynamics

$$\Delta x_f \propto U'_f(x_f) - p_f$$

$$\Delta W_{pkt} = (1-p)\frac{1}{W} - p\frac{W}{2}$$

$$\Delta W_{RTT} = \Delta W_{pkt} W = (1-p) - p\frac{W^2}{2} \cong 1 - p\frac{W^2}{2}$$

$$\Delta x = \frac{\Delta W_{RTT}}{RTT} = \frac{1}{RTT} - \frac{RTT}{2} p\, x^2$$

$$= \frac{RTT}{2} x^2 \left(\frac{2}{x^2 RTT^2} - p\right)$$

TCP/Reno Dynamics

$$\Delta x_f \propto U'_f(x_f) - p_f$$

$$\Delta x = \frac{RTT}{2} x^2 \left(\frac{2}{x^2 RTT^2} - p\right)$$

$$U'_f(x_f) - p_f$$

$$\Rightarrow U'_f(x_f) = \left(\frac{\sqrt{2}}{x_f RTT}\right)^2 \quad \Rightarrow U_f(x_f) = -\frac{2}{RTT^2 x_f}$$

TCP/Vegas Dynamics

$$\Delta x_f \propto U'_f(x_f) - p_f$$

$$\Delta w_{RTT} \approx -(w - xRTT_{min} - \alpha)$$

$$\Delta x = \frac{\Delta WRTT}{RTT} = -(\frac{w}{RTT} - \frac{x}{RTT}RTT_{min} - \frac{\alpha}{RTT})$$

$$= -\frac{w}{RTT} + \frac{x}{RTT}RTT_{min} + \frac{\alpha}{RTT}$$

$$= -x + \frac{x}{RTT}RTT_{min} + \frac{\alpha}{RTT}$$

$$= \frac{x}{RTT}(-RTT + \mathrm{RTT}_{\mathrm{min}} + \frac{\alpha}{x})$$

$$= \frac{x}{RTT}(\frac{\alpha}{x} - (RTT - \mathrm{RTTmin}))$$

$$\begin{aligned} \Delta W &\simeq \alpha - (W - \frac{RTT_{min}}{RTT}W) \\ &\simeq \alpha - (W - \frac{RTT_{min}}{RTT}xRTT) \\ &\simeq -(W - xRTT_{min} - \alpha) \end{aligned}$$

TCP/Vegas Dynamics

$$\Delta x_f \propto U'_f\left(x_f\right) - p_f$$

$$\Delta x = \frac{x}{RTT}\left(\frac{\alpha}{x} - (RTT - \mathrm{RTTmin})\right)$$

$$U'_f\left(x_f\right) - p_f$$

$$\Rightarrow U'_f(x_f) = \frac{\alpha}{x} \qquad \Rightarrow U_f(x_f) = \alpha \log(x_f)$$

Summary: TCP/Vegas and TCP/Reno

- Pricing signal is queueing delay $T_{queueing}$

$$x_f = \frac{\alpha}{T_{queueing}}$$

$$U'_f(x_f) = T_{queueing}$$

$$\Rightarrow U'_f(x_f) = \frac{\alpha}{x_f}$$

$$\Rightarrow U_f(x_f) = \alpha \log(x_f)$$

- Pricing signal is loss rate p

$$x_f = \frac{\alpha}{RTT\sqrt{p}}$$

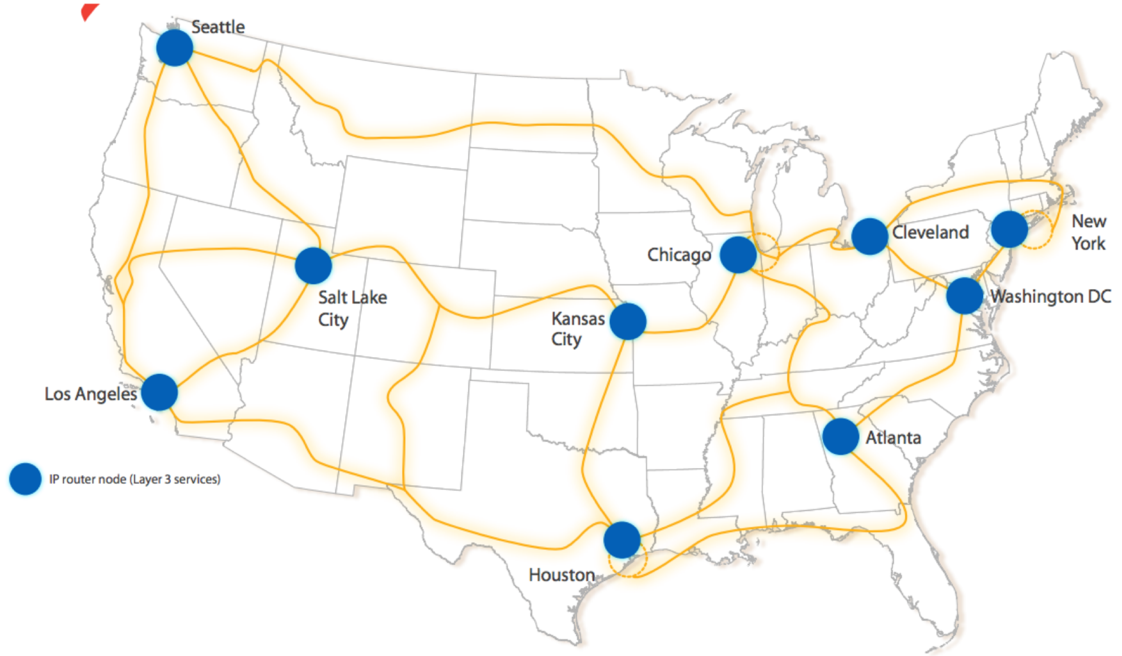
$$U'_f(x_f) = p$$

$$\Rightarrow U'_f(x_f) = \left(\frac{\alpha}{x_f RTT}\right)^2$$

$$\Rightarrow U_f(x_f) = -\frac{\alpha'}{RTT^2 x_f}$$

Discussion

- Assume that you are given a set of flows deployed at a given network topology.
- What is a simple way to predict TCP rate allocation?



Summary: Resource Allocation Frameworks

- Forward (design) engineering:
  - how to determine objective functions
  - given objective, how to design effective alg

$$
\begin{array}{ll}
\text{max} & \sum_{f \in F} U_f\left(x_f\right) \\
\text{subject to} & Ax \le C \\
\text{over} & x \ge 0
\end{array}
$$

- Reverse (understand) engineering:
  - understand current protocols (what are the objectives of TCP/Reno, TCP/Vegas?)
- Additional pointers:
  - http://www.statslab.cam.ac.uk/~frank/pf/