This draft fix a typo and illustrates the proof for the principle of **Inclusion-Exclusion**.

Part 1: Principle of Inclusion-Exlcusion (Fixed Typo in Slides page 20)

$$\left|\bigcup_{i=1}^{n} A_i\right| = \sum_{i=1}^{n} |A_i| - \sum_{1 \leqslant i < j \leqslant n} |A_i \cap A_j| + \sum_{1 \leqslant i < j < k \leqslant n} |A_i \cap A_j \cap A_k| - \cdots + (-1)^{n+1} |A_1 \cap \cdots \cap A_n| \tag{1}$$

Part 2: Proof (From Wiki[1])

Main idea: Element counting.

Choose an element contained in the union of all sets and let $A_1, A_2, \ldots, A_t$ be the individual sets containing it. (Note that *t* > 0.) Since the element is counted precisely once by the left-hand side of equation (1), we need to show that it is counted precisely once by the right-hand side. On the right-hand side, the only non-zero contributions occur when all the subsets in a particular term contain the chosen element, that is, all the subsets are selected from $A_1, A_2, \ldots, A_t$. The contribution is one for each of these sets (plus or minus depending on the term) and therefore is just the (signed) number of these subsets used in the term. We then have:

$$|\{A_i \mid 1 \leqslant i \leqslant t\}| - |\{A_i \cap A_j \mid 1 \leqslant i < j \leqslant t\}| + \cdots + (-1)^{t+1} |\{A_1 \cap A_2 \cap \cdots \cap A_t\}| = \binom{t}{1} - \binom{t}{2} + \cdots + (-1)^{t+1} \binom{t}{t}. \quad (2$$

By the binomial theorem,

$$0 = (1-1)^t = \binom{t}{0} - \binom{t}{1} + \binom{t}{2} - \cdots + (-1)^t \binom{t}{t}. \tag{3}$$

Using the fact that $\binom{t}{0} = 1$, and rearranging terms, we have

$$1 = \binom{t}{1} - \binom{t}{2} + \cdots + (-1)^{t+1} \binom{t}{t}, \tag{4}$$

and so, the chosen element is counted only once by the right-hand side of equation (1).

Reference

[1] https://en.wikipedia.org/wiki/Inclusion–exclusion_principle#Proof_of_main_statement